Physics 4335/8191(B)
Physics of Continuous Media
Fall 2012 Course Objectives

Fluids: See Fluids_Material.pdf handout for details.

1. **Pressure and hydrostatics:** Recognize the difference between contact forces and body forces. Know what type of field pressure is and how this can be proven. Recognize the equivalence between local and global formulations for hydrostatic equilibrium and how Gauss's theorem can be used to convert between the two.

2. **Surface tension:** Know about the molecular origins of surface tension, the contact angle, and capillary length. Be able to balance forces in hydrostatic problems that will allow you to find a solution for e.g. the shape of meniscus at a wall or the pressure excess in a sphere.

3. **Continuum dynamics:** What two basic mechanical equations govern the motion of continuous matter? Understand conservation laws and how they lead to equations of continuum dynamics. Understand what a material derivative is. Know the difference between a streamline, streakline, and path line. Cauchy's law of motion is important.

4. **Inviscid flow:** Recognize the types of problems where fluid can be considered to be inviscid, even if it is not explicitly stated that inviscid fluid mechanics applies. Understand when Bernoulli's theorem can be applied and use it to solve simple flow problems (e.g. Torricelli's law, Pitot tube). Solving for velocity "potential" as opposed individual velocity components is a useful tool for solving for flows that can be considered to be inviscid. Be familiar with the concept of the potential and stream functions and how they are related. Be able to solve for flows in simple geometries (e.g. around an infinite cylinder) using this formulation. Understand how these solutions for flow around some types of bodies lead to lift.

5. **Vorticity:** Know how to solve for vorticity and what the implications are for rotational and irrotational flow. Know how vorticity can be generated. Understand circulation and its relation to vorticity. Be able to solve for the pressure distribution in a vortex.

6. **Viscosity:** Understand the difference between boundary behaviour of viscous and inviscid flow. Be familiar with viscous friction and momentum diffusivity. Be familiar with the concept of the Reynolds number and how it can be used to determine the relative importance of terms in the nondimensionalized Navier-Stokes equations. Understand what is meant by flow reversibility for extremely viscous fluids.

7. **The Navier-Stokes equations:** For problems dealing with fluid motion, solutions to the Navier-Stokes equations (with appropriate boundary conditions) are used to describe the flow. Be able to apply the appropriate boundary conditions for various geometries (e.g. Poiseulle flow in a pipe, etc.) and solve for the Navier-Stokes equations. Recognize the conditions when particular terms of the N-S equations can be neglected.

8. **Dimensional analysis:** Why is it useful? Be able to use the Buckingham-Pi theorem to derive dimensionless numbers for describing the relevant physical parameters of a given system.

9. **Turbulence:** Understand the concept of turbulence, when it can develop, and the general differences between turbulent and laminar flow.

Elastic Solids: Chapters 6-9, 10 (sections 1& 2) of Lautrup plus the lecture notes.

1. **Stress Tensor:** Understand the origin of the stress tensor and the meaning of its components. For a body subject to external force density, $\underline{f}$ and internal stress tensor $\underline{\sigma}$, be able to determine the conditions for mechanical equilibrium (Cauchy's Equilibrium Equation). Know the appropriate boundary conditions for the components of the stress tensor.

2. **Displacement and Strain Tensor:** Understand the origin of the strain tensor, the meaning of its components, and its connection to the displacement field $\underline{u}(\underline{r})$ and the deformation gradient tensor $\underline{\nabla}\,\underline{u}$. Know the characteristics of strain for the special cases of linear shear, elongation, and pure dilation/compression. Be familiar with the geometric interpretations of the strain tensor in terms of the change in length of a "needle", and the change in magnitude of area and volume elements.

3. **Work of deformation and Hooke's law:** Know how to write the work done against external force density and internal stress in deforming an elastic material, and how to obtain Hooke's law for a linear, isotropic, elastic media. Be familiar with the linear elastic constants (E, K, λ, μ, and ν), the relations between them, and any physical constraints on them (e.g. the maximun and minimum values of the Poisson ration ν). Be able to use Hooke's law (or its inverse) to describe the relations between components of the stress and strain tensors for simple uniform deformations such as compression, shear, and elongation.

4. **Elastostatic Problems:** Know how use Hooke's law (or its inverse) to solve problems involving (i) shear-free settling of a body in a distributed external force field, (ii) pure, shear-free bending of a beam, (iii) uniform twisting of a beam, and (iv) compression of a symmetric body under pressure. Be familiar with the Euler–Bernouilli law relating the bending moment to the flexural rigidity of a beam uniformly bent into a shape with radius of curvature R. Be able to calculate the flexural rigidity of a beam given the shape of its cross section and its bulk elastic properties. Be familiar with the Coulomb–Saint-Venant law relating the twisting moment to the torsional rigidity of a beam uniformly twisted with torsion τ. Be able to calculate the torsional rigidity of a beam given the shape of its cross section and its bulk elastic properties.

5. **Slender Rods:** Know how to map the problem of bending of a long, thin symmetric rod onto the description of the trajectory of the centroid line. Be able to solve for the shape of such a slender rod under different loadings (e.g. distributed forces, forces applied to particular points, and moments applied to ends) and for different boundary conditions (e.g. free, forced, clamped). Know how to analyze the buckling of a slender rod under compression.

Viscoelastic Materials: Lecture notes

1. **Stress relaxation modulus:** Be familiar with the generalization to viscoelastic materials of Hooke's law relating the stress at time t in response to a static applied strain at $t = 0$; be able to use this relation to define the stress relaxation modulus $G(t)$. Be able to extend this approach to obtain the general linear stress response $\sigma(t)$ to a history of weak applied strains for $t' \leq t$. Be able to use $G(t)$ to calculate the effective viscosity η of a viscolelastic material.

2. **Complex Modulus:** Be familiar with the stress at time t in response to a weak oscillatory applied strain $\gamma(\omega t)$ of frequency ω; be able to use this to define the storage and loss moduli, $G'(\omega)$ and $G''(\omega)$, and the complex modulus $G^*(\omega) = G' + i\,G''$. Be able to compute $G^*(\omega)$ from a given stress relaxation modulus $G(t)$.

3. **Creep Compliance:** Be familiar with the generalization to viscoelastic materials of the inverse of Hooke's law relating the strain at time t in response to a static applied stress at $t = 0$; be able to use this relation to define the creep compliance $J(t)$. Be able to extend this approach to obtain the general linear strain response $\gamma(t)$ to a history of applied stresses for $t' \leq t$. Be able to compute the complex creep compliance $J^*(\omega)$ in response to an applied oscillatory stress from a given creep compliance $J(t)$.

4. **Viscoelastic Models:** Be familiar with the Maxwell model for a viscoelastic liquid and the Voigt model for a viscoelastic solid, and be able analyze such models to obtain differential equations relating $\gamma(t)$ and its time derivatives to $\sigma(t)$ and its time derivatives. For the the special cases of applied step stains or step stresses, be able to obtain $G(t)$ and $J(t)$ for these models.

5. **Non-Newtonian Pipe Flows*:** Grad students should be able to solve simple steady flow problems for non-Newtonian fluid flows in pipes or between parallel plates, given a model $\eta(\dot{\gamma})$ for the non-Newtonian fluid.