Assignment 1
Biophys 4322/5322

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1 Problem 1) Phillips 2.1

1.1 Part a)

Use Figure 2.1 to justify the assumption that a typical bacterial cell has a surface area of $6\mu m^2$ and a volume of $1\mu m^3$. Also express this volume in femtoliters. Make a corresponding estimate of the mass of such a bacterium.

We're only interested in knowing if the surface area $6\mu m^2$ and volume $1\mu m^3$ are reasonable based on Figure 2.1. The cell looks cylindrical with length $l = 2\mu m$ and diameter $1\mu m$ (radius $r = 0.5\mu m$). If these are normal values then the area of the cylinder (remembering the two end caps and remembering that $\pi \equiv 3$)

$$\begin{aligned} A &\approx 2\pi r l + \pi r^2 + \pi r^2 \\ &\approx \left[2 \times 3 \times 0.5 \times 2 + 2 \times 3 \times 0.5^2\right] \mu m^2 \\ &= [6 + 1.5]\, \mu m^2 \\ &= 7\mu m^2 \qquad (1) \\ V &\approx \pi r^2 l \\ &\approx 3 \times 0.25 \times 2\mu m^3 = 1.5\mu m^3 \\ &\approx 1\mu m^3, \qquad (2) \end{aligned}$$

which are both close enough to verify that $A \approx 6\mu m^2$ and $V \approx 1\mu m^3$.

To find this in femtoliters (fm) recall $1000L = m^3$ and femto $= 10^{15}$, which means

$$\begin{aligned} V = 1\mu m^3 &= 1\mu m^3 \left(\frac{m}{10^6 \mu m}\right)^3 \left(\frac{1000L}{1m^3}\right) \left(\frac{10^15 fL}{1L}\right) \\ &= 1fL. \qquad (3) \end{aligned}$$

How heavy does that make the cells? We know that cells aren't super buoyant but don't sink like rocks either. So let's say cells have the same density (ρ)

as water: $\rho_c \approx \rho_w \approx 1g/cm^3$. So then the mass of a single cell is approximately

$$\begin{aligned} M_c = V\rho_c \approx V\rho_w &= 1\mu m^3 \left(\frac{1g}{cm^3}\right) \\ &= \frac{\left(10^{-6}\right)^3 m^3}{\left(10^{-2}\right)^3 m^3} 1g = \frac{10^{-18}}{10^{-6}} g \\ &= 10^{-12} g = 1pg. \end{aligned} \tag{4}$$

1.2 Part b)

Roughly 2-3kg of bacteria are harboured in your large intestine. Estimate the total number. Estimate the total number of human cells in your body. Compare these numbers.

Let $M_{IL} = 2 - 3kg$. From the DGDs, we know to pick $3kg$. So the number of bacteria cells in your large intestine is

$$N_{LI} = \frac{M_{LI}}{M_c} \approx \frac{3kg}{10^{-12}g/\text{cell}} = 3 \times 10^{15}\text{cells} \tag{5}$$

Now to estimate the number of cells in my body. First off, human cells are much larger than E.coli. Let's say in general they are spherical with radius R and mass M_H. They aren't going to be a millimeter big (or even $0.1mm$) and I just said that they are bigger than E. coli; therefore,

$$\begin{aligned} 1\mu m <&R < 100\mu m \\ R &\approx 10\mu m \end{aligned} \tag{6}$$

$$\begin{aligned} M_H &\approx \rho V = \rho_w \frac{4}{3}\pi R^3 = 4\rho_w R^3 \\ &= \frac{1g}{1cm^3}\left(\frac{1cm^3}{10^{-6}m^3}\right) \times 4 \times \left(10^{-5}\right)^3 m^3 = 4 \times 10^{-9} g \\ &\approx 3 \times 10^{-9} g \end{aligned} \tag{7}$$

(remember only powers of 10 and 3 allowed when estimating). To know the number of human cells we need to know what I weigh. Just like in the DGDs we only want to deal with powers of 10 and with 3 so then

$$M_{\text{Tyler}} = 100kg,$$

which means that the number of human cells in my body is approximately

$$\begin{aligned} N_H \approx \frac{M_{\text{Tyler}}}{M_H} &= \frac{10^5 g}{3 \times 10^{-9} g} = \frac{1}{3} 10^{14} \\ &\approx 3 \times 10^{13}. \end{aligned} \tag{8}$$

So then we estimate that there are two orders of magnitude **more** bacterium in my large intestine than human cells in my body:

$$\frac{N_{LI}}{N_H} = \frac{3 \times 10^{15}}{3 \times 10^{13}} = 100. \tag{9}$$

Whoa.

1.3 Part c)

Some guy claims that there are 10^{28} prokaryotes in the first $200m$ of the world's oceans. If this is true than what is the total volume of all of them (in m^3 and in km^3) and what is there mean spacing *i.e.* how many cells are there per millimeter of water?

Let's say they have the same volume as the E. coli earlier in the question

$$V_p = 10^{-18} m^3. \tag{10}$$

Then the total volue of all 10^{28} of them is

$$\begin{aligned} V_T = NV_p &= 10^{28} 10^{-18} m^3 \\ &= 10^{10} m^3 \qquad (11) \\ &= 10^{10} m^3 \left(\frac{km}{1000m} \right)^3 = 10 km^3. \qquad (12) \end{aligned}$$

To estimate their mean spacing we have to know the volume available. Looking things up is not for respectable physicists; therefore, we recall having heard that 2/3 of the earth's surface is water and remember something about the size of the earth. Personally, I can only ever remember the radius from first year: $R_E = 6500 km$. So the total area of ocean A_O is approximately

$$\begin{aligned} A_O &\approx \frac{2}{3} 4\pi R_E^2 \\ &\approx 8 \times 6500^2 km^2 = 8 \times 6.5^2 \times \left(10^3\right)^2 (1000m)^2 \\ &\approx 320 \times 10^{12} m^2 = 3 \times 10^{14} m^2. \qquad (13) \end{aligned}$$

Then the volume of interest is

$$V = A_O \times 200m = 6 \times 10^{16} m^3, \tag{14}$$

which means that the number density of cells is

$$\begin{aligned} n = \frac{N}{V} &\approx \frac{10^{28}\text{cells}}{6 \times 10^{16} m^3} = \frac{1}{6} 10^{12} \frac{\text{cells}}{m^3} \\ &\approx \frac{10^{11}\text{cells}}{10^9 mm^3} = 100 \text{ cells}/mm^3 \qquad (15) \end{aligned}$$

2 Problem 2) Phillips 2.3

2.1 Part a)

Estimate the number of carbon atoms that constitute an E. coli.

According to page 35 of the textbook, 55% of the cell is proteins, 23% is nucleic acids, 9% is lipids and the other 15%-ish is other stuff. So let's just worry about those three main contributors. These numbers mean (page 9 of the textbook)

- 3×10^6 proteins
- 4×10^6 base pairs

- 2×10^6 lipids.

Protein How many amino acids are there in a protein? More than 1, more than 100 but probably not more than 1000. So that means there are 300. There are 5 carbons (hereafter, C) per amino acid. So then we know the number of carbons in the cell needed by the proteins

$$N_p = 3 \times 10^6 \times 300 \times 5C = 4.5 \times 10^9 C \approx 5 \times 10^9 C. \quad (16)$$

Nucleic Acid How many carbons per base pair (BP)? Answer: 4 nucleotides per BP and 5C per nucleotide, so 20C/BP which means

$$N_{NA} = 20C \times 4 \times 10^6 \quad (17)$$

$$= 8 \times 10^7 C. \quad (18)$$

Lipids How many carbons per phospholipid? About 40. so then

$$N_l \approx 40 \times 2 \times 10^7 C = 8 \times 10^8 C \quad (19)$$

Adding these three sources up we see the nucleic acids are pretty negligible and that the proteins by far contain the majority of carbon atoms

$$N_T = N_p + N_{NA} + N_l \approx 5 \times 10^9 C \quad (20)$$

2.2 Part b)

Ignoring the energy cost of synthesis, how many cells can be grown in a $5mL$ minimum culture before the carbon is exhausted?

We know there are $6C$ per sugar and from part a) we know that $5 \times 10^9 C$ are needed per cell; therefore,

$$N_S = \frac{5 \times 10^9}{6C} \approx 10^9 \quad (21)$$

sugars are needed to "create" a cell.

About the medium: $5mL$ at 0.2% glucose means that there is

$$5mL \times \frac{0.2g}{100mL} = \frac{1}{100} g = 10^{-2} g$$

of sugar present and 1 molecule of glucose is $C_6H_{12}O_6$ which means the weight of carbons is

$$\frac{180g}{6 \times 10^{23}\text{molecules}} = 2 \times 10^{-22} g/\text{molecule}$$

and so the total number of sugars present in the petri dish is

$$N_T = \frac{10^{-2}}{2 \times 10^{-22}}\text{sugars} = 5 \times 10^{19}\text{sugars}. \quad (22)$$

The number of carbons present divided by the number of carbons needed to constitute a cell gives the maximum possible number of cells that could be assembled from this many sugars to be

$$\frac{N_T}{N_S} = \frac{5 \times 10^{19}\text{sugars}}{10^9\text{sugars/cell}} = 5 \times 10^{10}\text{cells}. \quad (23)$$

3 Problem 4) Phillips 2.6

3.1 Part a)

Calculate the average volume and surface area of mitochondria in yeast based on Figure 2.9(D).

Consider the mitochondria to be a tube of length $h = 10\mu m$ and radius $r = 0.5\mu m$ so the area and volume area (don't forget the caps - just joking, they're not very important)

$$\begin{aligned} A &= 2\pi rh + 2\pi r^2 \approx 6r\,(h+r) \approx 6rh \\ &\approx 3 \times 10\mu m^2 = 20\mu m^2 \end{aligned} \tag{24}$$

$$\begin{aligned} V &= \pi r^2 h \approx 3\left(\frac{1}{2}\right)^2 10\mu m^3 \\ &= \frac{30}{4}\mu m^3 = 7.5\mu m^3 \\ &\approx 10\mu m^3. \end{aligned} \tag{25}$$

3.2 Part b)

Estimate the area of the endoplasmic reticulum (ER) if it is modeled as interpenetrating cylinders of diameter $d \approx 10nm$on a rectangular lattice of spacing $a \approx 60nm$.

Consider a "unit cell" of ER. 1/4 cylinders is along each edge of the cube (because each cylinder of a cell has to be shared with the four neighbouring cells. Since there are 12 edges on a cube the total number of cylinders n_c per unit cell is

$$n_c = \frac{1}{4}12 = 3. \tag{26}$$

Together these three cylinders have a surface area:

$$A = 3 \times 2\pi a(d/2) \approx 9ad. \tag{27}$$

This may feel a touch weird but we can write this as a sort of density, a surface area per unit volume, if you will:

$$\rho = \frac{A}{V} = \frac{9ad}{a^3} = \frac{9d}{a^2}. \tag{28}$$

Just like was done in the textbook, the total volume occupied by the ER is

$$V_{ER} = \frac{4\pi}{3}\left(R_{out}^3 - R_{nuc}^3\right) \approx 4\left(R_{out}^3 - R_{nuc}^3\right) \tag{29}$$

where $R_{nuc} = 5\mu m$ and $R_{out} = 10\mu m$. We can find the total surface area A_{ER}

by multiplying our surface area density ρ to the total volume V_{ER}:

$$\begin{aligned}
A_{ER} = \rho V_{ER} &\approx \frac{9d}{a^2} \times 4\left(R_{out}^3 - R_{nuc}^3\right) \\
&= 36 \frac{d\left(R_{out}^3 - R_{nuc}^3\right)}{a^2} \\
&= 36 \frac{10nm\left(10^3 - 5^3\right)\mu m^3}{\left(60nm\right)^2} = 36 \frac{10 \times 10^{-3}\mu m\left(10^3 - 125\right)\mu m^3}{3600 \times 10^{-6}\mu m^2} \\
&= \frac{36}{36}\frac{10}{10^{-4}}\frac{\mu m^4}{\mu m^2} \\
&= 10^5 \mu m^2
\end{aligned} \tag{30}$$